Student Name:________________________

CE 418.3 - Design in Reinforced Concrete

MIDTERM EXAMINATION

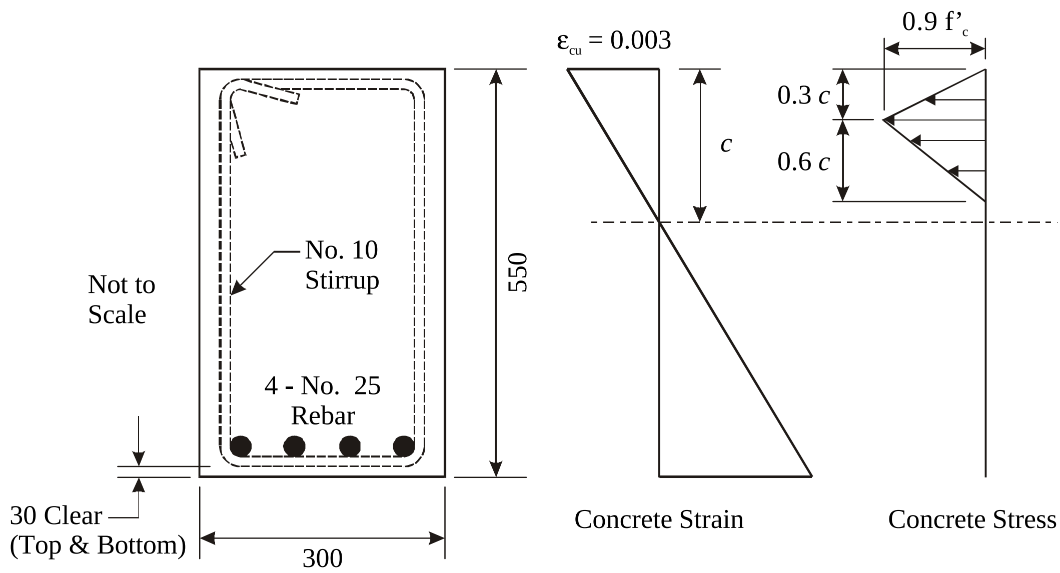
October 27, 2005

Time Allowed: 2 Hours Professor: B. Sparling

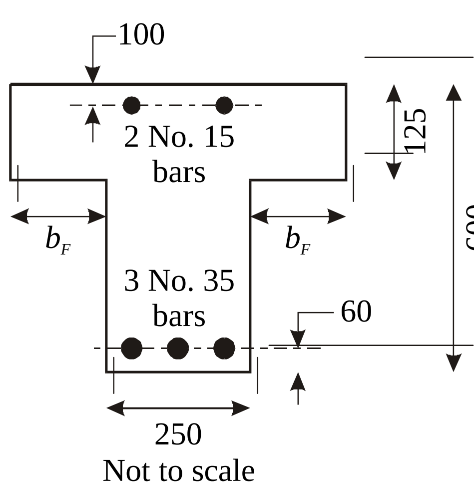
Notes:
- Closed book examination.
- CSA A23.3-04 is permitted along with rebar size and K_r vs. r sheets.
- Calculators may be used.
- The value of each question is provided along the left margin.
- Supplemental material is provided at the end of the exam (i.e. formulas).
- Show **all** your work, including all formulas and calculations.
- Clearly specify all assumptions made.
- All dimensions are in mm unless noted otherwise.

MARKS

26 **QUESTION 1:** As an alternative to the equivalent rectangular (Whitney) concrete stress distribution provided in CSA-A23.3-04, a new triangular stress distribution (shown below) has been proposed. Estimate the **nominal** (ideal) moment capacity M_n of the beam shown below based on the given strain and stress distributions. The beam is constructed using Grade 400 reinforcing steel and concrete with a design strength of $f_c' = 30$ MPa .



30 **QUESTION 2:** The doubly reinforced concrete beam shown below is subjected to a positive bending moment and constructed using **Grade 500** reinforcing steel and concrete with a design strength of $f_c' = 25$ MPa . In accordance with the limit states design method outlined in CSA A23.3-04, determine the width b_F of the outstanding flanges required to produce a tensile strain equal to $2.5\,\varepsilon_y$ (2.5 times the yield strain) in the principal tensile reinforcement at failure.



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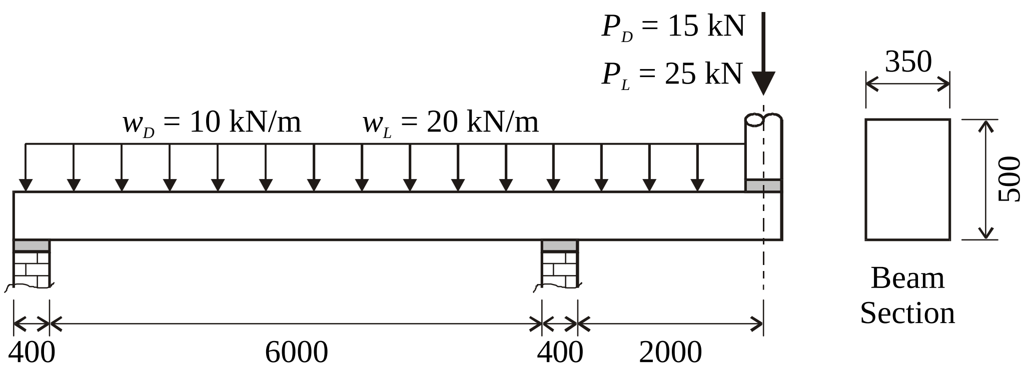
32 **QUESTION 3:** The reinforced concrete beam shown below is simply supported on two 400 mm wide masonry walls and cantilevers out 2 m past the right wall to support a steel post. The beam supports the uniformly distributed, superimposed specified loads (w_D and w_L) shown on the sketch, along with the specified concentrated loads (P_D and P_L) from the post; the uniformly distributed load is applied over the full length of the beam (i.e. no consideration of load patterns is required). The beam is constructed using Grade 400 reinforcing steel and concrete with a design strength of $f_c' = 30\text{ MPa}$; No. 10 stirrups are used, along with a concrete cover of 30 mm and a minimum clear spacing of 1.4 d_b between the main reinforcing bars.

In accordance with relevant requirements of CSA A23.3-04, select the principal flexural reinforcement as instructed below, assuming singly reinforced behaviour. Provide a sketch for each case showing the selected reinforcing steel.

(a) Using the K_r vs. ρ design table as an aid, select the positive principal reinforcement for the span between the masonry walls.

(b) Starting from the basic force and moment equilibrium equations, select the negative reinforcement required directly above the masonry wall on the right.



QUESTION 4: Provide brief answers to the following questions **in the space provided on this examination paper**. Answers in point form are acceptable. Sketches may be provided to supplement your answers where appropriate.

6 (a) List four of the most significant fundamental assumptions that serve as the basis for the analysis of beams at ultimate conditions.

6 (b) List four benefits derived from the use of compression reinforcement in doubly reinforced beams.

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Supplemental Material:

- **Material Properties:** $f_c = 0.65$ $\phi_s = 0.85$ $\alpha_D = 1.25$ $\alpha_L = 1.5$

$$f'_{ct} = \frac{t}{\alpha + \beta t}\ f'_c \qquad \frac{f_c}{f'_c} = 2\left(\frac{\varepsilon_c}{\varepsilon'_c}\right) - \left(\frac{\varepsilon_c}{\varepsilon'_c}\right)^2 \qquad f_{ct} = \frac{2P}{\pi d L} \approx 0.53\sqrt{f'_c}$$

$$E_c = \left(3300\sqrt{f'_c} + 6900\right)\left(\gamma_c/2300\right)^{1.5} \qquad E_s = 200{,}000 \text{ MPa} \qquad \varepsilon_{cu} = 0.0035$$

$$f_r = 0.6\,\lambda\sqrt{f'_c} \qquad \gamma_c = 2400 \text{ kg/m}^3$$

- **Flexural Analysis:** $\Sigma F_x = 0$ $\Sigma M = 0 \rightarrow M = T\,(j\,d) = C_c\,(j\,d)$ $C_c = T$

$$C_c = \int_0^c f_c\,\mathrm{d}A \qquad \bar{y}\,C_c = \int_0^c y\,f_c\,\mathrm{d}A \qquad C_c = (\phi_c\ \alpha_1\ f'_c)(\text{Area}) \qquad T = \phi_s\ A_s\ f_s$$

$$\alpha_1 = 0.85 - 0.0015\ f'_c \geq 0.67 \qquad \beta_1 = 0.97 - 0.0025\ f'_c \geq 0.67 \qquad a = \beta_1\ c$$

$$a = \frac{\phi_s\ A_s\ f_s}{\phi_c\ \alpha_1\ f'_c\ b} \qquad \varepsilon_s = \varepsilon_{cu}\left(\frac{d-c}{c}\right) \qquad \frac{c}{d} \leq \frac{700}{700 + f_y} \qquad \frac{d'}{c} \leq 1 - \frac{f_y}{700}$$

$$(A_s)_{\text{bal}} = \frac{\phi_c\ \alpha_1\ f'_c\,\beta_1\ b\ d}{\phi_s\ f_y}\left(\frac{700}{700 + f_y}\right) \qquad A_{s1} = A'_s\left(\frac{f'_s}{f_s} - \frac{\phi_c\ \alpha_1\ f'_c}{\phi_s\ f_s}\right) \qquad \varepsilon'_s = \varepsilon_{cu}\left(\frac{c - d'}{c}\right)$$

$$C_s = A'_s\left(\phi_s\ f'_s - \phi_c\ \alpha_1\ f'_c\right) \qquad A_{s2} = A_s - A_{s1}$$

$$M_{r1} = \phi_s\ A_{s1}\ f_{s1}\,(d - d') \qquad M_{r2} = \phi_s\ A_{s2}\ f_{s2}\left(d - \frac{a}{2}\right)$$

- **Flexural Design:** $A_{s_{\min}} = \dfrac{0.2\sqrt{f'_c}}{f_y}\ b_t\ h$ $\rho = \dfrac{A_s}{b\ d}$ $K_r = \dfrac{M_r \times 10^6}{b\ d^2}$

$$\rho_{\text{bal}} = \frac{\phi_c\ \alpha_1\ f'_c\,\beta_1}{\phi_s\ f_y}\left(\frac{700}{700 + f_y}\right) \qquad K_r = \phi_s\ \rho\ f_y\left(1 - \frac{\phi_s\ \rho\ f_y}{2\,\phi_c\ \alpha_1\ f'_c}\right) \qquad M_r \geq M_f$$

$$M_r = \phi_s\ \rho\ f_y\left(1 - \frac{\phi_s\ \rho\ f_y}{2\,\phi_c\ \alpha_1\ f'_c}\right) b\,d^2 \qquad \rho = \frac{\phi_c\ \alpha_1\ f'_c \pm \sqrt{(\phi_c\ \alpha_1\ f')^2 - 2\,K_r\,\phi_c\ \alpha_1\ f'}}{\phi_s\ f_y}$$

- **One-Way Floor Systems:** $A_{s_{\min}} = 0.002\ A_g$ $A_{sh} = \dfrac{(\phi_c\ \alpha_1\ f'_c)(h_F\ b)}{\phi_s\ f_y}$